METRON - *International Journal of Statistics*
2003, vol. LXI, n. 3, pp. 457-497

SATYA R. CHAKRAVARTY – PIETRO MULIERE

Welfare indicators: A review and new perspectives. 1. Measurement of inequality

Summary - The purpose of this paper is to present significant results on welfare theoretic approaches to income distribution based measurement problems. The topics covered are related to the measurement of inequality. Alternative forms of indices have been analyzed. The problem of ranking income distributions in terms of welfare, graphical techniques, different forms of equalizing transfers, stochastic dominance and inverse stochastic dominance have been studied extensively. Formal connections between these notions of orderings and dispersive ordering studied by statisticians is also discussed.

Key Words - Inequality; Indices; Welfare; Dominance.

1. Introduction

The two dimensions of an income distribution that spring to a layman's mind are the total and spread. That is, the size of the cake and how is it divided? Given, the population size, we ask the questions what is the mean income and how unequally are incomes distributed around the mean? Most people believe that given other things, a reduction in inequality should lead to an increase in the well-being of the society. However, there exists wide disagreement of views about how to measure inequality in an accurate way. Promotion of higher equality is an important issue in welfare economics. But traditional welfare economics does not offer much help so far as distributional issue is concerned (Sen (1973)). This probably explains why in empirical works some statistical measure of the dispersion of incomes is taken as an indicator of inequality. Although Dalton (1920) pointed out that the degree of inequality cannot be measured without introducing social judgements, Atkinson (1970), Kolm (1969) and Sen (1973) initiated the modern social welfare approach to inequality measurement. In the Atkinson-Kolm-Sen approach social judgements

Received September 2003 and revised November 2003.

We define, for $F \in \mathcal{F}$

$$F^{r+1}(t) = \int_0^t F^r(u)du \tag{2}$$

for all $t \in [0, \infty)$, where $r \geq 1$ is a positive integer. Obviously $F^1 = F$.

It is well known that:

$$F^r(t) = \frac{1}{(r-1)!} \int_0^t (t-y)^{r-1} dF(y)$$

for all $t \geq 0$. Analogously, we define:

$$H^{r+1}(t) = \int_0^t H^r(v)dv \tag{3}$$

for all $t \in [0, 1]$. Obviously $H^1(t) = H(t)$. $H^1(t)$ represents the income of the 100t-percent poorest individuals in the distribution X and is referred to as the quantile function. Thus, the mean income $\mu(X)$ can now be calculated as

$$\mu(X) = \int_0^1 H(t)dt\,.$$

Similarly, $H^2(t)$ represents the aggregate income possessed by the 100t-percent poorest individuals in the distribution X. The sequences $F^{r+1}(t)$ and $H^{r+1}(t)$ will assure us to define in Section 4 a sequence of stochastic dominances.

In this context a fundamental order is the Lorenz order. To define the Lorenz order, consider the Lorenz function $L_X : [0, 1] \to [0, 1]$,

$$L_X(t) = \frac{1}{\mu(X)} \int_0^t H(s)ds \tag{4}$$

with $0 \leq t \leq 1$.

The graph of the Lorenz function is the *Lorenz curve*. An inequality measure is a functional that assigns a real number to every income distribution. One of the most common measures of inequality is the Gini index, which is defined as:

$$G(X) = 1 - 2\int_0^1 L_X(p)dp = 1 - \frac{2}{\mu(X)} \int_0^1 \int_0^p H(t)\,dtdp\,. \tag{5}$$

As we pointed out in the Introduction it was suggested in the pioneering paper of Dalton (1920) that any measure of income inequality has an underlying social welfare function. Dalton's approach was developed further in Atkinson (1970). He defines the *equally distributed equivalent (EDE) income* of $X = (x_1, x_2, \ldots, x_n)$ to be that level of income which, if enjoyed by every individual,

Associativity

If F_1, F_2 and F_3 in $\mathcal{P}_A$ are such that

$$\mu_{\mathcal{J}}(F_1) = \mu_{\mathcal{J}}(F_2)$$

then for every $F_3 \in \mathcal{P}_I$ and $\lambda \in]0, 1[$

$$\mu_{\mathcal{J}}(\lambda F_1 + (1-\lambda)F_3) = \mu_{\mathcal{J}}(\lambda F_2 + (1-\lambda)F_3).$$

If, in the evaluation of welfare, we are indifferent between two distributions, this indifference is preserved if both distributions are mixed with a third distribution in the same proportions. In de Finetti's work, the characterisation of $\mu_{\mathcal{J}}(F)$ is the following:

Theorem 2.1. *Let A be a compact interval and let $\mu_{\mathcal{J}}$ be defined by (6). $\mu_{\mathcal{J}}$ satisfies strict monotonicity and associativity if and only if there exists a function u, continuous and strictly monotone, such that for every $F \in \mathcal{P}_A$*

$$\mu_{\mathcal{J}}(F) = u^{-1}\left(\int_A u(x)dF(x)\right) \tag{7}$$

where u is unique up to a positive affine transformation.

If $u(x) = x$ is chosen in (7), we get the the arithmetic mean, i.e. the mathematical expectation, μ_F of F.

The function $u(x)$ is interpreted as the individual utility function, and $u(\mu_{\mathcal{J}}(F))$ as the welfare index of $\mathcal{J}$. In a model of decision under risk, the quasi-linear mean in (7) corresponds to the expected utility index and u to the von Neumann–Morgenstern utility function. In this framework, Ramsey (1926) and von Neumann–Morgenstern (1947) provided different axiomatizations. For a survey see Muliere and Parmigiani (1993).

$$W(F) = u(\mu_{\mathcal{J}}(F)) = \int_A u(x)dF(x) = E(u(X))$$

expresses the social welfare of a society with income distribution F. If $u(x)$ is strictly concave, the individual utility function u increases at a decreasing rate, hence the social welfare increases when income is transfered from a richer to a poorer individual (for a review on welfare means, see Mosler and Muliere (1998)).

where c is a scalar such that $(X + c1^n) \in D^n$. Clearly, while in the former case income ratios are a source of envy, in the latter case people's feeling about deprivation due to higher incomes depends on absolute income differentials (see note 2). The classes of inequality indices satisfying invariance conditions (9) and (10) respectively may be rather large. Certain desirable properties can reduce the number of allowable indices. It has been argued in the literature that an inequality index $I : D \to R$, whether relative or absolute, should satisfy three postulates, namely, symmetry, the principle of population and the Pigou-Dalton transfers principle, which are stated below.

Symmetry (SYM)

For all $n \in N$, $X \in D^n$, $I^n(X) = I^n(Y)$, where Y is any permutation of X.

Symmetry means that inequality remains unchanged under any reordering of incomes. Under SYM any two individuals can trade their positions. One implication of symmetry is that we can define an index of inequality directly on ordered distributions.

Quite often we become interested in cross population comparisons of inequality. The following postulate, suggested by Dalton (1920), enables us to compare inequality over different population sizes.

Population Principle (POP)

For all $n \in N$, $X \in D^n$, $I^n(X) = I^{mn}(Y)$ where Y is the m-fold replication of X, that is, $Y = (x^{(1)}, x^{(2)}, \ldots, x^{(m)})$ with each $x^{(j)}$ being X.

According to POP, if a population is replicated several times, then the inequality levels of the original and the replicated populations are the same. In other words, POP views inequality as an average concept. Using replications, two distributions with different population sizes can be made to possess the same population size and POP keeps inequality unchanged under replications. It may be noted that POP is a property of all inequality indices that are defined on the continuum.

A third property which can be regarded as a central property of inequality indices is the Pigou (1912)-Dalton (1920) transfers principle.

The Pigou-Dalton transfers principle demands that a transfer of income from a person to anyone with a lower (higher) income should decrease (increase) inequality. We say that $X \in D^n$ is obtained from $Y \in D^n$ by a progressive transfer if there exist two persons i and j such that $x_k = y_k$ for all $k \neq i, j$; $x_i - y_i = y_j - x_j > 0$; $y_i < x_i < y_j$; and $y_i < x_j < y_j$. That is, X and Y are identical except for a positive transfer of income from person j to person i who has a lower income than j. Further, the transfer is such that it does not change the relative positions of the affected persons, that is, the donor of the transfer does not become poorer than the recipient. We can equivalently say that Y has been obtained from X by a regressive transfer.

Transfer Sensitivity (TRS)

For all $n \in N$, $Y \in D^n$, $I^n(X) < I^n(Y)$, whenever X is obtained from Y by a FACT.

Transfer sensitivity requires inequality to decrease under a FACT, which is composed of a progressive transfer and a regressive transfer, the former taking place at lower incomes than the latter such that the variance of the distribution does not change.

A positional version of the diminishing transfers principle is the *principle of positional transfer sensitivity*, requiring that a transfer from any person to someone who has a lower income, given that there is a fixed proportion of population between them, should attach more weight at the lower end of the distribution (see Mehran (1976), Kakwani (1980a) and Zoli (1999)).

Let $\Delta I^n_{i+t,i}(Y^*(\delta))$ be the reduction in inequality in Y^* due to a (rank preserving) progressive transfer of δ units of income from the person with rank $(i+t)$ to the person with rank i, where $t > 0$ is an integer.

Principle of Positional Transfer Sensitivity (PPT)

For all $n \in N$ and $Y^* \in D^n$ and for any pair of individuals i and j, $\Delta I^n_{i+t,i}(Y^*(\delta)) > \Delta I^n_{j+t,j}(Y^*(\delta))$, where $j > i$.

Note that for convenience PPT has been defined on ordered distributions. It implies that a combination of a (rank preserving) progressive transfer and a (rank preserving) regressive transfer of the same denomination, where the latter is taking place at higher incomes than the former reduces inequality.

It may be worthwhile to mention that recent experimental studies have not approved PDT unambiguously (see, for example, Amiel and Cowell (1992), Ballano and Ruiz-Castillo (1993) and Harrison and Seidl (1994)). This motivated several researchers to suggest weaker versions of PDT (see Eichhorn and Gehrig (1981), Castagnoli and Muliere (1990) and Mosler and Muliere (1996)). As weaker forms of PDT, Mosler and Muliere (1996) considered the principle of transfers about θ and star-shaped principle of transfers at θ, where θ may be a given constant, a function of mean income or a quantile of the income distribution.

Principle of transfers about θ

Given a fixed $\theta > 0$ and the non-identical ordered distributions $X^*, Y^* \in D^n$ with the same mean, we say that X^* has been obtained from Y^* by a sequence of transfers about θ if $x_i^* \leq \theta$ for $x_i^* - y_i^* \geq 0$, $x_i^* \geq \theta$ for $x_i^* - y_i^* \leq 0$.

That is, a transfer about θ is a rank preserving progressive transfer from a person with income above θ to someone who has a lower income than θ. For instance, the distribution (100, 480, 490) results from (100, 470, 500) by a transfer about $\theta = 490$ but not about $\theta = 470$.

I_D^n is bounded between zero and one, where the lower bound is achieved whenever the incomes are equal. This index tells us by how much (in relative terms) we can increase social welfare by distributing incomes equally. Since u is cardinal, it is necessary that I_D^n should remain invariant under affine transformations of u. But I_D^n does not satisfy this property. For a discussion of Dalton's approach see also Ferreri (1978, 1980), Benedetti (1980), Giorgi (1984, 1985) and Muliere (1987).

3.3. The Atkinson-Kolm-Sen approach and related issues

The form of social welfare function chosen by Dalton(1920) is quite restrictive. Therefore, following Sen (1973) we assume that ethical judgements on alternative distributions of income are summarized by the social welfare function $W : D \to R$, where W is ordinally significant. It is further assumed that for all $n \in N$, W^n is continuous, increasing and strictly S-concave. Continuity ensures that minor observational errors on incomes does not give rise to abrupt jump in the value of the social welfare function. Increasing-ness means that if we increase any income, keeping the remaining fixed, social welfare increases. Increasing-ness is analogous to the strong Pareto preference condition. Strict S-concavity, as we will see, demands that a rank preserving transfer of income from a person to anybody who has a lower income increases social welfare (see note 3). Given any $X \in D^n$, the Atkinson (1970)-Kolm (1969)-Sen (1973) equally distributed equivalent (EDE) income is defined as that level of income which if given to everybody will make the existing distribution X ethically indifferent (indifferent as measured by W^n). Thus, x_e is implicitly defined by

$$W^n(x_e 1^n) = W^n(X). \tag{17}$$

Given assumptions about W^n, we can solve (17) uniquely for x_e:

$$x_e = \mu_{\mathcal{J}}(X). \tag{18}$$

By continuity of W^n, $\mu_{\mathcal{J}}(X)$ is a continuous function. Furthermore $\mu_{\mathcal{J}}(X)$ is a specific numerical representation of W^n, that is,

$$W^n(X) \geq W^n(Y) \Longleftrightarrow \mu_{\mathcal{J}}(X) \geq \mu_{\mathcal{J}}(Y) \Longleftrightarrow x_e \geq y_e. \tag{19}$$

Thus, one income distribution is socially better than another if and only if its EDE income is higher. The indifference surfaces of $\mu_{\mathcal{J}}(X)$ are numbered so that

$$\mu_{\mathcal{J}}(c1^n) = c \tag{20}$$

with $c > 0$.

Then

$$\mu^r_{\mathcal{J}}(X) = \left[\frac{1}{n}\sum_{i=1}^{n} x_j^r\right]^{\frac{1}{r}} \tag{25}$$

or

$$\mu_{\mathcal{J}}(X) = \left[\prod_{i=1}^{n} x_j\right]^{\frac{1}{n}} . \tag{26}$$

Therefore the only linear homogeneous means are the power-mean and the geometric mean. Consequently, the class of power-mean and the geometric mean is characterized by reflexivity, strict monotonicity, associativity and linear homogeneity. For $r = 0$, we obtain the geometric mean, whereas for $r = 1$ we get the arithmetic mean, for $r = -1$ it becomes the harmonic mean. The parameter r determines the curvature of the social indifference surfaces. For any finite value of $r < 1$, the welfare contour becomes strictly convex to the origin and the degree of convexity increases as r decreases. As $r \to -\infty$, $\mu^r_{\mathcal{J}}(X) \to min_i(x_i)$ the Rawlsian maximin social welfare function (Rawls(1971)). On the other hand, as $r \to 1$, $\mu^r_{\mathcal{J}}(X) \to \mu(X)$, the mean income, which ignores distributional consideration and judges social welfare on the basis of size only. Therefore, if $r \leq 1$ (this means that u is concave) we obtain the inequality

$$x_e = \mu^r_{\mathcal{J}}(X) \leq \mu(X) .$$

The AKS index of inequality associated with the welfare function in (25) and (26) is the Atkinson (1970) index given by:

$$I^n_r(X) = 1 - \frac{1}{\mu(X)}\left[\frac{1}{n}\sum_{i=1}^{n} x_i^r\right]^{\frac{1}{r}} \tag{27}$$

or

$$I^n_r(X) = 1 - \frac{1}{\mu(X)}\left[\prod_{i=1}^{n} x_i\right]^{\frac{1}{n}} . \tag{28}$$

I^n_r satisfies TRS for all values of $r < 1$. For a given X, I^n_r is decreasing in r. As the value of r decreases greater weight is attached to transfers at the lower end of the profile. As $r \to -\infty$, $I^n_r \to 1 - min_i(\frac{x_i}{\mu(X)})$, the relative maximin index, which corresponds to the maximin criterion.

An alternative of interest arises from the Gini social welfare function $\mu_G : D \to R$, where for all $n \in N$, $X \in D^n$,

$$\mu_G(X) = \frac{1}{n^2}\sum_{i=1}^{n}(2(n-i)+1)x_i^* \tag{29}$$

for a fixed number of persons $(j-i-1)$ between the donor j and the recipient i, a progressive transfer is valued more by these indices if the transfer occurs at lower income levels. That is, they satisfy PPT. In contrast, for the Gini index the reduction due to the same progressive transfer depends on the difference $(j-i)$, which shows that given the difference $(j-i)$, the Gini index is sensitive to transfers in the same way whether they take place at the top of the income distributiòn or they concern low incomes and hence it fails to demonstrate positional transfer sensitivity. (For a comparison of Bonferroni index and Gini index in term of social welfare see Benedetti (1986).) However, one major shortcoming of the two Bonferroni indices is that they violate the principle of population and this makes them unsuitable for comparison of inequality across different-sized populations, but the Gini index is suitable in this context (see note 5).

It is therefore clear that to every homothetic social welfare function, there corresponds a different index of inequality and vice-versa. For instance, we can derive welfare functions associated with the Theil(1967) entropy index and the coefficient of variation. These indices will differ depending on the corresponding social welfare functions.

The concept of absolute inequality was introduced by Kolm (1976, 1976a). Blackorby and Donaldson (1980) made a detailed investigation on the properties of the social welfare functions associated with alternative absolute inequality indices. The Blackorby-Donaldson-Kolm (BDK) index of inequality is defined by $A_{\text{BDK}} : R_+ \to R$, where for all $n \in N$ and $X \in R^n_+$,

$$A^n_{\text{BDK}} = \mu(X) - \mu_J(X)\,. \tag{33}$$

A_{BDK} is continuous, strictly S-convex and bounded from below by zero, where this bound is achieved whenever incomes are equal. It satisfies POP if $\mu_{\mathcal{J}}(X)$ satisfies the same. It gives the per capita income that could be saved if society distributed incomes equally without any welfare loss. It also determines the size of absolute welfare loss associated with the existence of inequality.

Since in (33) two functions appear in a difference form, it is reasonable to regard A^n_{BDK} as an absolute index. A^n_{BDK} is an absolute index if and only if $\mu_{\mathcal{J}}$ is unit- translatable, that is

$$\mu_J(X + c1^n) = \mu_J(X) + c \tag{34}$$

for all X and c, where c is a scalar such that $(X + c1^n) \in R^n_+$. Since $\mu_J(X)$ and W^n are ordinally equivalent, unit translatability of $\mu_{\mathcal{J}}(X)$ means that W^n is translatable (see Blackorby and Donaldson (1980) and Chakravarty (1990)) (see note 6). From policy point of view the absolute index determines the total cost of per capita inequality in the sense that it tells us how much must be added in absolute terms to the income of every member in an n-person society his subgroup's EDE income (see Blackorby, Donaldson and Auersperg (1981)) (see note 7).

Some welfare functions are both homothetic and translatable. Such welfare functions are called distributionally homothetic (Blackorby and Donaldson (1980)) (see note 8). Examples are the Gini and Bonferroni welfare functions. We can therefore generate both relative and absolute indices from such welfare functions. For instance, using the Gini and Bonferroni welfare functions in (33) we get the Gini and Bonferroni absolute indices which are given respectively by:

$$A^n_{\mathrm{G}}(X) = \mu(X) - \frac{1}{n^2}\sum_{i=1}^{n}(2(n-i)+1)x_i^* \tag{38}$$

and

$$A^n_{\mathrm{B}}(X) = \mu(X) - \frac{1}{n}\sum_{i=1}^{n}\mu_i\,. \tag{39}$$

An inequality index of this type is called a compromise index-when its relative form is multiplied by the mean income we get an absolute index and conversely, if the absolute version is divided by the mean income the resulting index becomes relative. (For further examples of such indices, see Ebert (1988b) and Chakravarty (1990)).

Remark 4.1. A comparison between utility based indices is possible using a comparison between $\mu_{\mathcal{J}}$. Let $\mu_{u_1}(F)$ and $\mu_{u_2}(F)$ be quasi-linear means with two different utility functions u_1 and u_2. If u_1 is increasing then

$$\mu_{u_1}(F) \geq \mu_{u_2}(F)$$

holds for every F if and only if $u_1 \circ u_2$ is convex.

Remark 4.2. Chakravarty and Dutta (1987) proved that distributionally homothetic social welfare functions become useful for measuring economic distance between two income distributions. The economic distance between two populations is supposed to reflect the degree of affluence or well-being of one population relative to another. Hence this rules out a simple comparison of the inequality of incomes within respective populations, since this approach neglects the differences in mean incomes and so ignores an important factor which influences the relative well-being of two populations.

Assuming that $\mu_{\mathcal{J}}$ is population replication invariant, Chakravarty and Dutta (1987) characterized

$$|\mu_{\mathcal{J}}(X) - \mu_{\mathcal{J}}(Y))| \tag{40}$$

well-being. Contributions along this line have come from Maasoumi (1986, 1999), Tsui (1995,1999), Dardanoni (1995), Koshevoy and Mosler (1996, 1997), Bourguignon (1999) and others.

Suppose that the well-being of a person depends on k attributes. Let x_{ij} be the quantity of attribute j possessed by person i. Assuming that there are only two attributes, Bourguignon (1999) considered the following CES type individual utility function:

$$U(x_{i1}, x_{i2}) = (\alpha_1 x_{i1}^{-\beta} + \alpha_2 x_{i2}^{-\beta})^{-\frac{1+\gamma}{\beta}} \tag{44}$$

where $-1 < \gamma < 0$ is the inequality sensitivity parameter and β represents the degree of substitutability between the two attributes. A natural multidimensional extension to the Dalton index is then:

$$I_{\mathrm{D}}(C) = 1 - \frac{\sum_{j=1}^{n}(\alpha_1 x_{i1}^{-\beta} + \alpha_2 x_{i2}^{-\beta})^{-\frac{1+\gamma}{\beta}}}{n(\alpha_1 \mu_1^{-\beta} + \alpha_2 \mu_2^{-\beta})^{-\frac{1+\gamma}{\beta}}} \tag{45}$$

where C is the matrix showing quantities of the two attributes possessed by different individuals and μ_1 and μ_2 are the means of attributes 1 and 2 respectively. We now consider an issue which is of very much practical importance in multidimensional measurement. Redistributing the two attributes so as to keep the marginal distributions constant and increase the correlation between them should increase or decrease inequality according as the attributes are substitutes or complements, that is, the cross derivative u_{12} is negative or positive. In terms of the parameters of the utility function this condition becomes negativity or positivity of $\beta + 1 + \gamma$. By strict quasi-concavity of u, $\beta > -1$ and $\gamma < 0$. Values of these parameters can now be chosen appropriately to ensure increasing or decreasing inequality under a correlation increasing switch that keeps marginal distribution constant. It may be noted that the index in (45) is quite close to an index of multidimensional inequality suggested by Maasoumi (1986). Analogous extensions of the Atkinson and Kolm-Pollak indices to the multidimensional framework was developed by Tsui (1995).

The central idea underlying the inequality-welfare relationship is that social welfare should be an increasing function of mean income (efficiency) and a decreasing function of inequality. Evidently, alternatives to the formulations considered above are possible. For example, we can define social welfare function as

$$W^n(X) = \mu e^{-I(X)}. \tag{46}$$

Another possible formulation is

$$W^n(X) = \frac{\mu}{(1 + I^n(X))}.$$

if

$$L_X(p) \geq L_Y(p)$$

for all $p \in [0, 1]$, with $>$ for some p.

That is, the Lorenz curve of X is nowhere below that of Y and strictly above at some places(at least). (An axiomatic characterization of the Lorenz ordering can be found in Aaberge (2001).) By scaling up the Lorenz curve of a distribution by its mean income, we get the generalized Lorenz curve of the distribution. Formally, the generalized Lorenz curve of X is defined as

$$GL(X, p) = \mu(X)L_X(p) .$$

Definition 4.2. We say that X generalized Lorenz dominates Y, $X \geq_{GL} Y$ for short, if

$$GL(X, p) \geq GL(Y, p)$$

for all $p \in [0, 1]$ with $>$ for some p.

If the means of the distributions are the same, the Lorenz and generalized Lorenz dominations coincide.

Atkinson (1970) made use of the formal similarity between the ranking of income distributions and the ranking of probability distributions in terms of expected utility. In particular, Atkinson used results from Rothschild and Stiglitz (1970) to demonstrate equivalence between Lorenz domination and second order stochastic dominance. To discuss these results formally, we first define stochastic dominance.

Definition 4.3. Given any two income distributions X and Y with distribution functions F_X and F_Y, we say that X-rth order stochastic dominates Y, which we denote by $X \geq_r Y$, if

$$F_X^r(t) \leq F_Y^r(t) \tag{47}$$

for all $t \in [0, \infty]$ with$<$ for at least one t, where r can be equal to any finite positive integer.

Thus, for first order stochastic dominance between X and Y we need inequality between the corresponding distribution functions.

Similarly, X second order stochastic dominates Y if we have

$$F_X^2(t) \leq F_Y^2(t)$$

for all $t \in [0, \infty]$ with $<$for some t.

The condition $X \geq_r Y$ is equivalent to the requirement that the expected utility under F_X is greater than that under F_Y, where all odd order derivatives dominates Y, then X is regarded as more equal than Y by all inequality indices that fulfil symmetry and the Pigou-Dalton condition. The converse is true as well. However, if the two curves cross we can get two inequality indices satisfying PDT and SYM that disagree on the ranking of the two distributions.

Since Theorem 4.1 relies on constancy of mean income and the population size, its scope is quite limited, it is inapplicable to comparisons of inequality and welfare of distributions with variable means and population sizes. The following theorem due to Kolm (1969), Marshall and Olkin (1979) and Shorrocks (1983) shows that using the generalized Lorenz curve we can rank distributions with different means over a fixed population size.

Theorem 4.2. *Let $X, Y \in D^n$ be arbitrary. Then the following conditions are equivalent:*

(a) $X \geq_{GL} Y$.

(b) $X \geq_2 Y$.

(c) $\sum_{i=1}^{n} u(x_i) > \sum_{i=1}^{n} u(y_i)$ *for all utility functions $u : J \rightarrow R$ that are increasing and strictly concave, where J is some interval in the non-negative part of the real line.*

(d) $W^n(X) > W^n(Y)$ *for all increasing and strictly S-concave social welfare functions W^n.*

Thus, Theorem 4.2 says that of two distributions X and Y over a given population, X is regarded as better than Y by all increasing and strictly S-concave social welfare functions if only if X generalized Lorenz dominates Y. This in turn is equivalent to the condition that X second order stochastic dominates Y. Theorem 4.2, however, does not tell us anything about inequality ranking of the concerned distributions. Inequality ranking here cannot be obtained by condition (a). To understand this, suppose that X is obtained from Y by increasing the income of the richest person. Then $X \geq_{GL} Y$. But in this case X is also regarded as more unequal than Y by all relative inequality indices that fulfil SYM and PDT (see Chakravarty (1990)). In fact, the following theorem of Foster (1985) (see also Fields and Fei (1978) and Chakravarty (1990)) shows that the appropriate technique here is the Lorenz criterion.

Theorem 4.3. *Let $X, Y \in D^n$ be arbitrary. Then the following conditions are equivalent:*

(a) $X \geq_L Y$.

(b) $I^n(X) < I^n(Y)$ *for all relative inequality indices I^n that satisfy SYM and PDT.*

We can also focus our attention on fixed mean, arbitrary population size case. In this case the domain of definition of the inequality index is an appropriate subset D_c, where $D_c = \{X \in D | \mu(X) = c\}$. The following theorem distance evaluated at p between the line of equality and $GL(X, p)$. It gives the average amount of income necessary to make everyone's (among bottom p proportion of population) income equal to the current mean income. Replacing $X \geq_L Y$ by $X \geq_{AL} Y$, the absolute Lorenz dominance, which we define in the same way as the Lorenz ordering, in part (a) of Theorems 4.3 and 4.6 and 'relative' by 'absolute' in part (b) of the theorems, we get absolute counterparts to Theorems 4.3 and 4.6. If we say that X second order absolute stochastic dominates Y, whenever the distribution $[X - \mu(X)1^n]$ second order stochastic dominates the distribution $[Y - \mu(Y)1^n]$, then a condition analogous to (c) in Theorem 4.6 can be developed as well.

In practice, the Lorenz curves of income distributions are often found to intersect and hence the Lorenz ordering of the concerned distributions turns out to be inconclusive in such cases. Therefore, though the Lorenz domination and second degree stochastic dominance have appealing normative justifications, they have the serious problem of being inconclusive in many practical situations. Hence it may be necessary to appeal to the third degree stochastic dominance as a ranking criterion. Shorrocks and Foster (1987) proved the following analogue to Theorem 4.4 in this context.

Theorem 4.7. *Let $X, Y \in D_c$ be arbitrary. Then the following conditions are equivalent:*

(a) *There exist replications U and V of distributions X and Y respectively such that U and V have the same population size and U^* can be obtained from V^* by a finite sequence of rank preserving progressive transfers and/or FACT.*

(b) *$X \geq_3 Y$, that is, X third order stochastic dominates Y.*

(c) *$I(X) < I(Y)$ for all inequality indices $I : D_c \to R$ that fulfil SYM, POP, PDT and TRP.*

Theorem 4.7 shows that third degree stochastic dominance is necessary and sufficient for unanimous ranking of two income distributions by all transfer sensitive inequality indices.

Although for intersecting Lorenz curves the inequality ranking of distributions by indices identified in condition (e) of Theorem 4.4 is not conclusive, it is possible to obtain an indisputable ordering for intersecting Lorenz curves under special circumstances when we restrict attention to transfer sensitive indices.

The variance of the distributions plays a crucial role here (Shorrocks and Foster (1987)). More precisely, when the Lorenz curve of X intersects that of Y *once from above*, then a sufficient condition for X to be preferred to Y by the third order stochastic dominance criterion is that the variance of X is lower than that of Y (Shorrocks and Foster (1987)).

Given $X, Y \in D$, $L_X(p)$ is said to intersect $L_Y(p)$ once and from above if there exists $p^* \in (0, 1)$ such that $L_X(p) > L_Y(p)$ for all $p \in (0, p^*)$ and where $v(p) \geq 0$ is the weight attached to the income of the person with rank p. $W_Y(F)$ increases under a progressive transfer if and only if $v(p)$ is decreasing (Yaari (1988)). Similarly, for PPT to hold it is necessary and sufficient that $v(p)$ is strictly convex (Mehran (1976)). In fact, dominance in terms of the Yaari social welfare function corresponds to inverse stochastic dominance introduced by Muliere and Scarsini(1989).

Definition 4.4. Given two income distributions X and Y with distribution functions F_X and F_Y respectively, we say that X-rth order inverse stochastic dominates Y, which we write $X \geq_r^{-1} Y$, if

$$H_X^r(p) \geq H_Y^r(p) \tag{52}$$

for all $p \in [0, 1]$ with $>$ for some p, where r is any arbitrary finite positive integer.

The orderings $\geq_r^{-1}$ form a sequence of progressively finer partial orderings:

$$X \geq_r^{-1} Y \rightarrow X \geq_s^{-1} Y \tag{53}$$

with $s \geq r$.

That is, $\geq_s^{-1}$ orders all pairs of distributions that are ordered by $\geq_r^{-1}$ and some more. Thus as we pass from $\geq_r^{-1}$to $\geq_{r+1}^{-1}$ each of the previously performed comparisons between pairs of distributions remains valid, and some more are included. It is easy to see that the direct first order stochastic dominance and the first order inverse stochastic dominance are equivalent. In fact, under equality of means equivalence holds for second order dominance as well. When $r \geq 3$ the equivalence does not hold anymore (Muliere and Scarsini (1989).)

We formally state this as

Theorem 4.9. *Let X and Y be two income distributions with the same mean μ. Then the following conditions are equivalent:*

(a) $X \geq_2 Y$.

(b) $X \geq_2^{-1} Y$.

(c) $X \geq_L Y$, *that is, the Lorenz curve of X dominates that of Y.*

Theorem 4.9 gives a normative justification of the inverse second order dominance in terms of Lorenz ordering. But, as stated equivalence of the type given by (a) and (b) does not carry over beyond second order. However, Zoli (1999) established the following normative significance of the inverse third order dominance:

When g is the identity function we get the symmetric quasi-linear mean. For $\psi(t) = t$ and $g(p) = 1-(1-p)^\delta$, where $\delta > 1$, we obtain the Donaldson-Weymark (1980, 1983) single parameter Gini social welfare function

$$W_\delta(X) = -\int_0^\infty td[1 - F_X(t)]^\delta . \tag{55}$$

Therefore, if $X \geq_{r+1} Y$, then $W_\delta(F_X) \geq W_\delta(F_Y)$ for $\delta \geq r$. For $\delta = 2$, W_δ becomes the Gini welfare function. The higher is the value of the single parameter δ, the closer are the implicit ethics to the maximin rule (see also Bossert (1990) and Aaberge (2000,2001)). Assuming that ψ is the identity function and substituting $t = H(p)$ we note that $W(X)$ in (54) becomes W(F) in (51). Hence the Gini and Yaari welfare functions can be interpreted as rank dependent quasi-linear means.

Social welfare functions, which rely on the Lorenz divergence function and can be represented as rank dependent quasi-linear means, have also been suggested in the literature. An example is

$$W_g(X) = \mu(X)(1 - I_g(X)) \tag{56}$$

where X is any arbitrary income distribution, $g : [0, 1] \to [0, 1]$ is continuous, increasing, $\int_0^1 g(p)dp = 0$ and

$$I_g(X) = \int_0^1 (p - L_X(p))dg(p) . \tag{57}$$

$I_g(X)$ is a weighted area between the diagonal line and the Lorenz curve, and can be regarded as an index of inequality. Boundedness of g ensures that $I_g(X)$ is also bounded between zero and one. Increasingness of g is necessary and sufficient for PDT. The normalization $\int_0^1 g(p)dp = 0$ guarantees that $I_g(X)$ achieves its lower bound zero if everybody receives the same income.

If $g(p) = 2p - 1$, we get the Gini welfare function and Gini inequality index in (56) and (57) respectively. On the other hand, for $g(p) = 3p(2 - p)$, the corresponding welfare and inequality indices become the ones suggested by Mehran (1976) (see Nygard and Sandstrom (1981) and Mosler and Muliere (1998), for further discussion).

Although increasingness of g ensures PDT for $I_g(X)$, there is no guarantee that TRS will hold. The following generalization of the Gini index suggested by Chakravarty (1988) avoids this shortcoming:

$$I_\phi(X) = 2\phi^{-1}\left[\int_0^1 \phi(p - L_X(p))dp\right] \tag{58}$$

($n = 3$), couples with children, married couples without child and single person households. Assume that the households have been arranged in non-increasing order of needs. The income utility function for household of type i is denoted by u_i. Let u_{AB} be the class of all utility profiles $(u_1, u_2, \dots, u_n)$ satisfying the following conditions:

(a) each u_i is increasing and strictly concave.
(b) $(u_i^{'} - u_{i+1}^{'})$ is positive and decreasing in i.

The following theorem of Atkinson and Bourguignon (1987) can now be stated:

Theorem 4.13. *Let there be two societies with income distribution functions F_X and F_Y respectively. Suppose that the social welfare function W is additive across types, with utility function u_i from profiles u_{AB} being applied to different types. Then the following conditions are equivalent:*

(a) $W(F_X) > W(F_Y)$ *for all utility profiles* $(u_1, u_2, \dots, u_n) \in u_{AB}$.
(b) *There is generalized Lorenz dominance of F_X over F_Y in each of the sub-populations comprising the j most needy groups,* $j = 1, 2, , \dots, n$.

The procedure is to take the neediest group first, then add the second neediest group, and so on until all groups are included, checking at each stage for generalized Lorenz domination. Obviously at the terminal stage of the sequential generalized Lorenz dominance we need conventional generalized Lorenz dominance. See also, Atkinson (1990), Jenkins and Lambert (1993), Ok and Lambert (1999) and Ebert (2000) (see note 11).

So far we have presented our discussion in terms income distributions. There exists formal connections between inequality ordering and dispersive ordering. A dispersive ordering is a partial ordering of distributions according to their degree of dispersion (see, Shaked (1982), Lynch, Mimmack and Proschan (1983)).

Definition 4.7. A distribution function F_X is said to be less dispersed than another distribution function F_Y if

$$H_X(\beta) - H_X(\alpha) \leq H_Y(\beta) - H_Y(\alpha) \quad (60)$$

for all $0 < \alpha < \beta < 1$ and we denote this by $F_X \leq_{disp} F_Y$.

That is, the income gap between 100βpercent poorest individuals and 100αpercent poorest individuals is not higher under F_X than that under F_Y.

The following theorem of Shaked (1982) and Lynch, Mimmack and Proschan (1983) shows that dispersive ordering is equivalent to the condition that some functions, determined by the pair of the underlying distributions, change sign at most once. This is also equivalent to first order stochastic dominance in the weak sense. Formally, we have:

5. Concluding remarks

Ethical index number theory provides a way to link social indicators of inequality to the moral judgements required for policy decisions. As we have seen, the advantage of welfare indicators over descriptive ones is that the value judgements that are employed in both cases become explicit in the former. We also discuss a method for uncovering ethical judgements implicit in the application of descriptive indices to policy decisions. The literature on dominance which says how one distribution can be preferred to another on welfare ground is also surveyed extensively.

6. Notes

(1) See, for instance, Kakwani (1980), Ebert (1988), Chakravarty (1990, 1999), Cowell (1995, 2000), Foster and Sen (1997), Silber (1999), Blackorby, Bossert and Donaldson (1999), Lambert (2001), and Dutta (2002).

(2) Bossert and Pfingsten (1990) developed a more general notion of inequality equivalence using a convex mix of relative and absolute concepts (see also Zoli (2003)).

(3) Equivalently, $W^n : D^n \rightarrow R$ is called S-concave if

$$W^n(BY) \geq W^n(Y)$$

for all Y and for all bistochastic matrices B of order n. An $n \times n$ non-negative matrix B is called a bistochastic matrix if each of its rows and columns sums to one. Strict S-concavity requires strict inequality whenever BY is not a permutation of Y. A function $G^n : D^n \rightarrow R$ is called S-convex (strictly S-convex) if $-G^n$ is S-concave (strictly S-concave). All S-concave and S-convex functions are symmetric.

(4) Formally, W^n is called homothetic if it can be written as $\psi(\widetilde{W}^n(X))$, where ψ is increasing in its argument and $\widetilde{W}^n$ is linear homogenous.

(5) For further discussion on the Bonferroni index, see Tarsitano (1990), Giorgi and Mondani (1994,1995), Giorgi (1998), Giorgi and Crescenzi (2001a, 2001b, 2001c).

(6) Formally, W'^n is called translatable if it can be written as $\psi(\widehat{W}^n(X))$,where ψ is increasing in its argument and $\widehat{W}^n$ is unit translatable.

(7) On related matters, see Bhattacharya and Mahalanobis (1967), Bourguignon (1979), Cowell (1980, 1995), Cowell and Kuga (1981), Shorrocks (1980, 1984, 1988), Foster (1983), Ebert (1988a, 1999), Silber (1989), Lambert

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SATYA R. CHAKRAVARTY
Economic Research Unit
Indian Statistical Institute
203 B.T. Road, Calcutta
700108 (India)
satya@isical.ac.in

PIETRO MULIERE
Istituto Metodi Quantitativi
Università L. Bocconi
viale Isonzo, 25
20135 Milano (Italy)
Pietro.Muliere@uni-bocconi.it